Abstract — This article introduces a model for piezoresistive CMOS cantilevers, taking into account thermal phenomena. The case study device is a magnetometer. Two thermal effects are considered: the sensitivity of gauges to temperature and thermal actuation. The model is then exploited for the development of electro-thermal test strategies. Test techniques based on step response and harmonic response to electrically-induced thermal stimuli are studied and experimentally validated.

Index Terms— MEMS, modeling, thermal, test.

I. INTRODUCTION

Many MEMS devices make use of thermal effects for their functionality: thermal imager [1], thermally-actuated micro-mirror [2], AFM cantilever [3]. In other cases, thermal phenomena are parasitic effects that must be cancelled to guarantee the correct behavior of the sensor, but can be exploited for test purposes [4,5,6,7]. In both cases, a model taking into account thermal behaviors is necessary during the design phase. In CMOS suspended structures such as cantilevers, bridges or membranes, polysilicon piezoresistive gauges are often used to sense stress in the structure. By this way a force applied to the frame is measured. However, if an element dissipates heat in the structure, thermally-based effects may be observed. First, the gauges are sensitive to the temperature. Second, the structural materials that compose the mechanical frame have different thermal expansion coefficients. So, a change of temperature translates in a bending of the frame and the so-obtained stresses are then detected by the gauges.

In this paper, we are introducing a model for a CMOS magnetometer, which takes into account the thermal effects. The proposed model has been validated against silicon and it is then demonstrated that the thermal parasitic signal can be exploited to test the integrity of the mechanical frame. Basically, the proposed test procedure consists in observing if the temperature of the structure is correct and if the structure can be actuated by a thermally-induced stimulus. These structural tests have been implemented experimentally and have demonstrated a good ability to detect under-etching defects, and most of the catastrophic faults that may affect the sensor. The main advantage of this test technique is to be low-cost because the calibrated stimuli are induced electrically and, in consequence, can be generated by standard test equipments and even on-chip. In the presented test case, we have demonstrated that it is possible to test a magnetometer without a magnetic field. This test procedure is also convenient to add a BIST function in the sensor design and, by this way, to reduce the cost of the test equipment. It is worth noting that electrically-induced thermal stimuli are a good alternative in the case of most of CMOS MEMS where electrostatic force can not be used for testing because of the large gaps left by the etched silicon [8].

The paper is organized as follow. First, sensor background is introduced. Then, thermal coupling is studied and a model taking into account these effects is developed and validated against silicon. In the last section, the application for test is considered taking into account under-etching defects. Fault simulation results are analyzed and compared to experimental results, which confirm that the thermal stimuli are suitable to detect this type of defects.

II. SENSOR BACKGROUND

A. Technology and sensing principle

The MEMS magnetometer under study is based on an U-shaped mechanical frame attached to the die at two anchor points, as illustrated in Fig. 1. Its operating principle relies on the interaction between the magnetic field B to be measured and a known current I flowing through a planar coil embedded in the frame. The resulting Lorentz force (Fl) actuates the structure. The stress induced in the structure near the anchor points is measured by means of embedded piezoresistive gauges. These gauges are placed in a Wheatstone bridge with two reference resistors located above the substrate in order to transform the resistance variation into a voltage variation.

![Fig. 1 U-shaped cantilever: (a) schematic view and (b) SEM photograph](image)

The U-shaped structure has been fabricated with a front side bulk micromachining MEMS process. This process is based on a standard 0.8 µm CMOS process and wet etching of the obtained dies allows releasing suspended mechanical structures composed of the back-end layers of the CMOS process. Experimental characterization of the device has demonstrated that the mechanical structure reacts as a second-order low-pass system with a first resonant mode...
around 22.5 kHz and a large quality factor of about 100. Obviously to obtain the best sensor sensitivity, the structure should be actuated at this particular resonant frequency in order to benefit from the high quality factor. Therefore, the operating mode of the sensor involves an AC bias current $I$ applied to the aluminum coil at the mechanical resonant frequency; the resulting AC voltage at the output of the Wheatstone bridge is then processed by a conditioning electronics to extract the magnetic field.

B. Mechanical Modeling

The mechanical model of the sensor is described in Fig. 2. The current $I_f$ is created by the applied voltage $U_f$. The mechanical behavior of the cantilever can be represented in the electrical domain by a 2nd order circuit with an inductance, a capacitance and a resistor corresponding to the spring ($K$), mass ($M$) and damper ($D$) parameters of the cantilever. The Lorentz force $F_L$ is considered to be applied at the extremity of the cantilever. The resulting displacement $z$ is related to the equivalent static force $F_e$ with $z = \frac{F_e}{K}$ and the relative change of resistance in the gauge writes:

$$\frac{\Delta R}{R} = K_R \times \frac{z}{K} = \frac{z}{K}$$

where $K_R$ represents a gauge coefficient that depends on the piezoresistivity of polysilicon, geometry and the spring constant of the cantilever.

These two phenomena can be experimentally observed from the harmonic response of the device (Fig. 3). If a differential sine wave voltage is applied to the coil, corresponding to an excitation current $I$ equal to:

$$I = I_0 \sin(2 \pi f \cdot t)$$

The resulting power dissipated in the coil is then:

$$P = P_{f_0} = P_{f_c} \cdot 1.5(1 - \cos(2 \cdot 2 \pi f \cdot t))$$

where $\frac{P_{f_c}}{P_{f_0}}$ is the portion of the resistance of the coil $R_f$ located on top of the structure and thus participating to the temperature variation. The power dissipated is then composed of a static power dissipation plus a sinusoidal one at twice the frequency of the input voltage.

A peak appears at 12 kHz, which is half the resonant frequency of the cantilever. This peak clearly corresponds to the thermal actuation of the cantilever at its mechanical resonance. Except around this peak, the measured signal is due to the sensitivity of the gauge and is a linear function of the temperature variations. The response can be approximated to the response of a first order system and the response is fully characterized by the parameters in the table below.

### III. THERMAL MODEL

#### A. Experimental response of the sensor to a thermal stimulus

The excitation current $I$, flowing into the coil of the cantilever, dissipates Joule power. As the cantilever is suspended in a large cavity, its thermal isolation w.r.t. the substrate is high. It results a temperature rise of the structure that has two consequences:

- The first consequence is a direct effect of the temperature on the gauges. The resistance of the gauges changes due to their thermal sensitivity. As the reference resistors are placed on the substrate that acts as a thermal sink, their resistance doesn’t change. As a result, the Wheatstone bridge becomes unbalanced and an electrical signal is induced at its output.

- The second consequence is a thermal actuation of the cantilever because it is composed of several materials with different thermal expansion coefficients. As the cantilever is actuated, stresses are induced in the piezoresistive gauges thus leading to an unbalanced wheatstone bridge and to an electrical output signal.
B. Modeling the thermal behavior

In order to calculate the temperature of the cantilever for a given power dissipation, we assume that convection and radiation phenomena are negligible compared to heat conduction along the beam. Moreover considering the geometry of the structure, the temperature gradients in the y and z directions could be neglected. The system is then equivalent to the longitudinal beam of Fig. 4. We note $T(x)$ the temperature rise (relative to the ambient temperature) at the location $x$. It is null at the anchors because the substrate acts as a thermal sink.

\[ T(0) = 0 \]
\[ T(L_b) = 0 \]

![Fig. 4 One dimensional representation of the cantilever](image)

As the power is dissipated on the entire cantilever, an easy way to solve the problem is to build a distributed thermal model (Fig. 5) with an equivalent electrical circuit. The beam is divided in $n$ sections composed of a power source ($\Delta P$), a thermal resistance ($\Delta R$) and a thermal capacitance ($\Delta C$) representing respectively the local power dissipation, the thermal conduction along the beam and the portion of the volume to be heated. The input of the system is the total dissipated power $P_f$. The outputs are the temperatures at the nodes.

![Fig. 5 Distributed thermal model](image)

This model can be easily implemented in a simulation tool using a mixed HDL language. It can be automatically generated as a function of $n$, with the parameters calculated as follows:

\[ \Delta P = \frac{P_f}{n} \]  
\[ \Delta R = \frac{L_b}{n} \sum_{i=1}^{n} W_i \cdot e_i \]  
\[ \Delta C = \sum_{i=1}^{n} \left( C_i \cdot D_i \cdot e_i \cdot W_i \right) \cdot L_b / n \]  

where $W_i$, $e_i$, $K_i$ and $C_i$ are respectively the width, the thickness, the thermal conductivity and the specific heat of each material $i$ (values are taken from [9]).

Obviously $n$ must be high enough to have accurate simulation results. We have determined that with $n$ equal to 39, the error on the estimated maximum temperature is smaller than 0.5%. This model permits to estimate the temperature of the gauges through simulation. The gauges being much smaller than the cantilever, it can be assumed that the average temperature of one gauge ($T_g$) is equal to the temperature at its center. Due to the symmetry of the structure, the temperature of both gauges is equal and $T_g = T[4] = T[35]$ with $n$ equal to 39. The harmonic response of the system, considering ($T_g$) is given on Fig. 6.

![Fig. 6 Simulated harmonic response using the distributed model](image)

Once again the system can be approximated by a first order system and we can introduce the compact model of thermal heating given in Fig. 7, in which the equivalent thermal resistance ($R_{th}$) and capacitance ($C_{th}$) are defined as follows:

\[ R_{th} = \frac{T_g}{P_f} \]  
\[ C_{th} = \frac{1}{R_{th} \cdot 2 \pi \cdot F_c} \]  

where $F_c$ is the -3dB cut off frequency.

![Fig. 7 Equivalent electrical circuit of the thermal heating](image)

Theoretical and experimental values of these parameters are summarized in Table 3. Experimental values are calculated from Table 2. It is worth noting that the proposed model has been also validated with respect to IR images [10].

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Theoretical value</th>
<th>Experimental value</th>
<th>Unit</th>
</tr>
</thead>
<tbody>
<tr>
<td>$R_{th}$</td>
<td>1,707</td>
<td>1,547</td>
<td>°C/W</td>
</tr>
<tr>
<td>$C_{th}$</td>
<td>2.63</td>
<td>2.7</td>
<td>µJ/°C</td>
</tr>
</tbody>
</table>

Table 3: Theoretical and experimental values of $R_{th}$ and $C_{th}$

C. Modeling of thermally-induced electrical signals

As it has been experimentally observed, the temperature variations in the cantilever result in two phenomena. The direct effect of temperature on the output signal of the sensor is due to the thermal sensitivity of the gauges ($K_{th} = 0.9 \cdot 10^{-3}$ for a polysilicon gauges), that relates the relative variation of the resistance to the gauge temperature: $\Delta R_{direct} / R = K_{th} \cdot T_g$  

The other effect is due to thermal actuation. As it has been introduced in [11] and verified against experimental results, the thermal expansion creates an equivalent force applied to the extremity of the structure ($F_{th}$) that is proportional to the temperature of the structure. So a coefficient as used in [5], [11] and [12], $K_d$ links $F_{th}$ to $T_g$ such as:

\[ F_{th} = K_d \cdot T_g \]  

This force creates a displacement $z_{th}$ of the extremity of the cantilever. Out of the resonance, this thermally-induced deformation results in a different shape of the bending compared with the magnetically-induced one. In case of thermal actuation the curvature is maximum at the extremity.
of the U-shaped frame (where the temperature is maximum), whereas in case of magnetic actuation the curvature is maximum at the anchor where the gauges are located [11]. However, when the cantilever resonates at its fundamental mode, the deformation does not depend on the actuation source. So the relation (1) that links the change of resistance to the displacement of the extremity is still valid when the frequency of the thermal signal is equal to the mechanical resonant frequency. As the thermal actuation phenomenon is experimentally observed only at the resonance, the value of $K_d$ at the resonance needs to be considered in the model.

D. Complete model

In the equivalent electrical circuit of the complete sensor model (Fig. 8), the force applied to the extremity of the cantilever is the sum of the magnetic one and the thermal one. The gauge resistance variation results from both the piezoresistive effect and the thermal sensitivity effect.

$$K_d$$ is empirically calculated by fitting the peak due to thermal actuation simulated and the experimental response of Fig. 3. Finally, the obtained model has been validated against silicon (Fig. 9).

![Complete model of the sensor](image)

Fig. 8 Complete model of the sensor

Kd is empirically calculated by fitting the peak due to thermal actuation simulated and the experimental response of Fig. 3. Finally, the obtained model has been validated against silicon (Fig. 9).

![Experimental thermal actuation](image)

Fig. 9 Harmonic response of the actual cantilever and its model

IV. APPLICATION FOR TEST

A. Fault modeling

With respect to the two-step fabrication process, defects may be related to either CMOS defects, i.e. spot defects affecting the back-end layers, or post-process defects. On one hand, polysilicon and aluminum may be affected by shorts, opens or bridges. On the other hand, some defects may affect the electro-mechanical structures during silicon etching. In that case, the main defect is under-etching that leads to non- or badly-released mechanical structures. As an illustration, Fig. 10.a shows a sample where the two cantilevers have been completely released while Fig. 10.b shows a sample where the mechanical frames are still connected to the substrate through pyramidal-shaped structures. Obviously, cantilevers of this type that are not fully-released will not operate.

![SEM photograph of a released cantilever and a badly-released cantilever](image)

Fig. 10 SEM photograph of (a) a released cantilever and (b) a badly-released cantilever

Now regarding the thermal behavior, it is clear that such defects have also an impact on the thermal balance of the cantilever. As indicated on Fig. 10.b, heat is conducted to the substrate through the pyramidal-shaped structures. Such defects can be modeled as a resistive short to the substrate with a thermal resistance $R_{def}$, as described in [12]. Fault injection can be easily performed in our distributed thermal model, simply adding the defect thermal resistance at the desired location (Fig. 11). It is then possible to evaluate how the temperature of the structure is affected for different defect sizes by varying the value of the defect thermal resistance.

![Fault injection in the distributed thermal model](image)

Fig. 11 Fault injection in the distributed thermal model

As an illustration, we have considered a defect located at the corners of the U-shaped structure. Fig. 12.a reports the variation of the gauge temperature as a function of defect size when a 5V voltage is applied to the coil. It can
be observed that the temperature of the gauges varies from 10.9K for a fully-released structure (Rdef = 1.10^8 K/W) down to 3.1K for a perfect thermal short to the substrate (Rdef = 0.1 K/W). From this curve, we can estimate the size of the smallest detectable defect. Indeed if we assume that normal process scattering on the mechanical structure and the material properties can result in a variation of 10% in the value of the thermal resistances of the model, it comes that the smallest detectable fault has a resistance of 100,000 K/W. We can relate the value of the thermal resistance of the pyramidal-shaped structure to the size of the defect with:

\[
R_{\text{def}} = \frac{1}{(K_{\text{Si}} \cdot a)}
\]

where \(K_{\text{Si}}\) is the thermal conductivity of the silicon substrate and \(a\) is the length of the square section that connects the cantilever. The smallest detectable resistance then corresponds to a defect that connects the cantilever with a section of 0.06µm. This is really small compared to the overall size of the structure and so, we can expect that most of the defects are detected by measuring the temperature of the gauge. In contrast, we can observe in Fig. 12.b that the gain-bandwidth product is not significantly affected by the defect whatever its size.

The analysis conducted using the distributed thermal model has permitted to identify the thermal behavior of defective structures. The next step is to perform fault injection at higher level considering the complete model of Fig. 8. This can be easily achieved by adding a resistor \(R_{\text{def}}\) in parallel with \(R_{th}\). By this way, the temperature of the gauge is affected while the gain-bandwidth product is preserved, which corresponds to the simulation results obtained with the distributed thermal model.

The defect also impacts the mechanical behavior of the structure. In fact, defects are equivalent to mechanical anchors attached to the corners of the U-shaped structure. In this situation, the structure can be viewed as composed of three bridges. The resonant frequency of these bridges is much higher than the one of the fully-released cantilever and their sensitivity to magnetic field and thermal actuation much lower. As a consequence no resonance peak due to thermal actuation can be observed. The fault can be injected in the model by forcing the force \(F\) to zero.

B. Test method

The complete model of the sensor together with its associated fault injection capabilities allows us to simulate the thermal behavior of fault-free and faulty devices. We can then investigate through simulation the efficiency of different test methods.

First, we have considered a step signal applied to the coil. In this situation, the thermal sensitivity of the gauge is the predominant effect. Fig. 13 presents simulation results obtained for fault-free and faulty devices with \(R_{\text{def}}^* = 682K/W\). At low frequency and more specifically for frequencies lower than the thermal cut-off frequency, distinction between the faulty device and the fault-free one is possible. The difference between the two responses is due to the thermal conduction through the defect, as in the case of a step voltage stimulus. At higher frequency, except at half the resonant frequency, the amplitude of the output signal is not affected by the presence of the defect. This is due to the conservation of the gain-bandwidth product. At half the resonant frequency, the thermal actuation peak is observed but only for the fault-free device.

The second technique we have considered involves harmonic response analysis. It consists in monitoring the amplitude of the sensor output when a sinusoidal input voltage is applied to the coil, for different frequencies. Fig. 14 presents simulation results obtained for fault-free and faulty devices with \(R_{\text{def}}^* = 682K/W\). At low frequency and more specifically for frequencies lower than the thermal cut-off frequency, distinction between the faulty device and the fault-free one is possible. The difference between the two responses is due to the thermal conduction through the defect, as in the case of a step voltage stimulus. At higher frequency, except at half the resonant frequency, the amplitude of the output signal is not affected by the presence of the defect. This is due to the conservation of the gain-bandwidth product. At half the resonant frequency, the thermal actuation peak is observed but only for the fault-free device.

To summarize, detection of faulty devices through harmonic response analysis is possible in two different frequency ranges: at low-frequency due to the thermal sensitivity of the gauge and at half the resonant frequency due to thermal actuation. From a practical point of view, there is no particular interest in implementing harmonic response analysis at low frequency because step response analysis provides similar information but necessitates less demanding resources. On the other hand, it may be interesting to implement harmonic response analysis around half the resonant frequency because it involves a different mechanism, i.e. thermal actuation. The idea is then to scan the input frequency around half the resonant frequency and to check the presence of the thermally-induced peak. Taking into account that the resonant frequency may vary of 30% due to process scattering, the input signal frequency should be scanned between 7.5 KHz and 15 KHz. In addition, the frequency step should be small enough to detect the presence of the peak, e.g. 50Hz in our case since the
thermally-induced peak exhibits a bandwidth of 200Hz. Obviously, this test solution based on harmonic response analysis is more time-consuming and costly than the solution based on step response analysis but it permits to verify the mechanical resonance of the device.

Finally, it is worth noting on Fig. 14 that the peak due to thermal actuation can be clearly distinguished from the one due to magnetic actuation. This is because the frequency of the thermal signal is twice the one of the input signal. By this way, the measurement of the thermal actuation signal is not distorted by the magnetic field environment and there is no need to monitor it.

C. Experimental results

To experimentally validate the test solutions investigated in the previous section, we are in possession of four types of device:

- Devices of type A are fully-released devices.
- Devices of type B are non fully-released devices that are still connected to the substrate through pyramidal-shaped structures but only at some tiny points.
- Devices of type C are non fully-released devices that are strongly connected to the substrate through bigger pyramidal-shaped structures.
- Devices of type D have not been etched at all; the structure is completely merged in silicon.

The four types have been experimentally analyzed considering step response and harmonic response analysis. Note that a low-noise on-chip amplifier is integrated with the sensor to provide a good resolution.

Fig. 15 reports the transient response measured for the different types of device when a voltage step of 2.3V is applied to the coil (the voltage step is lower than 5V to prevent the on-chip amplifier from saturation). It is obvious that this electro-thermal stimulus is sufficient to detect faulty devices of type C and D as output voltages exhibit significant differences. However it is not possible to distinguish between responses of type A and B devices. This situation corresponds to the case where only some tiny contacts exist between the cantilever and the un-etched silicon; the thermal resistance of the defect is too high to significantly affect the thermal conduction.

Table 4 summarizes the step response results. The gain-bandwidth product conservation is respected for type A, B and C devices. The type D device exhibits a higher gain-bandwidth product that is certainly due to the augmentation of the mass of the merged structure.

The harmonic response measured for the four types of device is reported on Fig. 16. As expected at low frequency, we can draw similar observations as for the step response analysis. The harmonic response is more interesting around half the resonant frequency because it permits to distinguish the hard-to-detect fault on the type B device. As illustrated on the close-up view of Fig. 16, the peak due to thermal actuation is not present for device B. We can consequently deduce that the cantilever can not move and the device is non-operating.

Finally, it is worth noting that the harmonic response may also be used to detect parametric faults on a well-released device. Indeed, the resonant frequency, the bandwidth and the amplitude of the resonance of the cantilever can be calculated from this analysis. These parameters can be useful to detect an abnormal behavior of the sensor.

V. CONCLUSION

In this paper, we have proposed a complete model for a piezoresistive cantilever that takes into account parasitic thermal effects. The variation of the frame temperature in function of the dissipated power on the mechanical structure can be calculated from low-level parameters such as thermal conduction of the materials and the geometry. Thermal actuation phenomenon has been also modeled thanks to an empirical parameter.

This model has been used to evaluate, by fault injection, two test methods to detect a catastrophic under-etching defect. The principle is based on the use of parasitic thermal signals as stimuli. Two effects have been exploited:

- The temperature of the structure that is affected by an under etching defect.
- Thermal actuation that disappears when a device is not completely released.

The first effect is associated to a step response analysis and the second one by an harmonic response. Step response is faster and allows classification of the size of the fault. However it is not sufficient to detect faulty devices that are
weakly connected to the substrate. Harmonic response thus increases the fault coverage but is more time consuming and its implementation is more complex. Finally, by giving information on the resonant frequency of the sensor, harmonic response may also be used to detect parametric faults of released devices.

VI. REFERENCES


