

# A Study of Flicker Noise in MOS Transistor Under Switched Bias Condition

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## ABSTRACT

This manuscript examines in detail the mechanisms and behavior of flicker noise in switched biased MOS transistors. Firstly, the PSD of a DC biased transistor is deduced using only Shockley-Read-Hall (SRH) statistics and the autocorrelation formalism. Then the analysis is extended, by means of simulations and using simple physical hypotheses, to a switched bias condition. The results allow explaining several reported experimental data. Particularly, the  $1/f$  form of flicker noise at very low frequencies is observed in simulations.

**Index Terms:** flicker noise switched biased MOS.

## 1. INTRODUCTION

Flicker noise or simply  $1/f$  noise is such that its power spectral density (PSD) varies with frequency in the form:

$$S(f) = K / f^\gamma \quad (1)$$

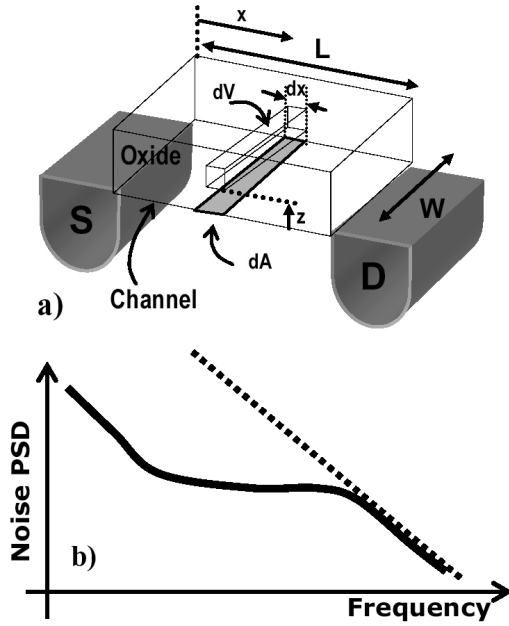
with  $K$ ,  $\gamma$ , constants, and  $\gamma \approx 1$ . It is quite well accepted that the sources of low frequency noise are mainly carrier number fluctuations due to random trapping–detrapping of carriers in energy states, named ‘traps’, near the surface of the semiconductor. From some time ago, switched biasing has been proposed as a technique for reducing the flicker noise itself in MOSFET’s [1]. An intuitive explanation of the phenomenon is that periodically turning ‘off’ the transistor’s channel, periodically forces a significant fraction of occupied traps to a known empty state, thus introducing some ‘order’ in the random process. A switched MOSFET flicker noise PSD resembles the plot in Figure 1.b [2,13]. Usual  $1/f$  spectrum is seen at frequencies greater than the switching frequency. At lower frequencies the noise (log scale) increases with a much smaller slope. Finally at an even lower frequency, the slope resembles again the original  $1/f$  spectrum.

Several authors proposed models to explain this particular behavior [3,4,12] however, the exact mechanism and the statistics of the switched noise current, are not yet clear. Particularly, reported models [3] pre-

dict a plateau at lowest frequencies that do not correctly address experimental results [2,13]. The model presented in [12] is not simulation based as [3,4] and shows a different approach at this problem. The goal of this paper is to discuss in detail flicker noise in a switched MOS transistor.

Let us first examine the DC bias case: consider a MOS transistor, and a small channel element of differential area  $dA = W \cdot dx$  as in Figure 1.a. Defects inside and at the surface of the oxide generate localized states (traps with energy  $E_t$ ), which may be occupied by carriers from the channel. Electrons (and holes) in the channel may tunnel to, and back from, these traps in a random process thus generating a noise current.  $N_A$  [ $\text{m}^{-2}$ ] will denote the number of occupied traps per unit area in the whole oxide volume above the channel element  $dA$ . The relation between the carrier densities in the channel named  $N$  [ $\text{m}^{-2}$ ], and  $N_A$  is given by the Reimbold’s coefficient  $r$  [5][10]. To find the drain current noise, the impact on  $I_D$  of local  $N$  fluctuations is integrated along the channel [5,6]. Thus a physics based flicker noise model should begin finding an expression for  $S_{N_A}(f)$  (the PSD of  $N_A$ ).

This paper is organized as follows: in section II, an explicit analytic deduction of  $S_{N_A}(f)$  (non-switched transistor) is presented using SRH statistics and the autocorrelation formalism. In section III the study is extended using simulations to examine the switched bias flicker noise.



**Figure 1:** a) a channel element  $dA$ , and oxide volume  $dV$  definition. b) Trapping-detrapping of carriers by oxide traps above  $dA$ , produce a noise current which in the case of a switched bias transistor, approximates the PSD of the plot.  $f_c$  switching frequency.

## 2. DEDUCTION OF DC BIASED FLICKER NOISE

To compute  $S_{N'_V}(f)$  we start by defining a small volume  $\Delta V = W \cdot dx \cdot dz$  as in Figure 1.a.  $N_t, N'_V [eV^{-1} \cdot m^{-3}]$  are respectively, the volume density of traps and occupied traps, inside  $\Delta V$ , and for a small energy interval  $E \leq E_t \leq E + \Delta E$ .  $f_t$  is the probability of a single trap to be occupied (which can be calculated in terms of the Fermi level of the system [7]) and  $t_{ox}$  is the thickness of the oxide.

To find the PSD of a random variable (i.e.  $N'_V$ ), it is necessary to compute the Fourier transform of its autocorrelation defined as:  $\Re(s) = \overline{\delta N'_V(t) \cdot \delta N'_V(t-s)}$ . In a time interval  $dt$  occupied traps may release their electron with a probability  $e_0$ . Empty traps may be occupied with a probability  $(c_0 n_S) \cdot dt$ , where  $n_S$  is the electron density in the channel: the denser the electron population in the conduction band is, the more likely it is that an electron would tunnel to the empty trap. Given an initial  $N'_V$  density of occupied traps, their expected variation in the time interval  $dt$  is written using SRH:

$$dN'_V = [c_0 n_S (N_t - N'_V) - e_0 N'_V] \cdot dt \quad (2)$$

$(N_t - N'_V)$  is the number of empty traps per unit volume. At equilibrium, the average  $N'_V$  must be kept constant so  $c_0 n_S (1 - f_t) - e_0 f_t = 0$ . But  $N'_V$  is not at equilibrium in (2);  $N'_{veq} = f_t N_t$  denotes the equilibrium value, which suffers variations:  $N'_V = N'_{veq} + \delta N'_V$ . If variations of  $n_S$  with  $N'_V$  are neglected it follows:

$$\frac{d(\delta N'_V)}{dt} = - (c_0 n_S + e_0) \delta N'_V \quad (3)$$

(3) is a first order differential equation with the solution

$$\delta N'_V = \delta N'_V|_{t=0} \cdot e^{-\frac{|t|}{\tau}} \quad (4)$$

Where  $\tau = \frac{1}{c_0 n_S + e_0}$ . Note that  $\delta N'_V$  is a random variable,  $\delta N'_V|_{t=0}$  is an arbitrary known initial condition; the absolute value in (4) was introduced for symmetry. To find the autocorrelation of the process it is necessary to integrate in all possible  $\delta N'_V$  taking into account the probability  $p(\delta N'_V)$ :

$$\Re(s) = \int_{-\infty}^{\infty} \delta N'_V \cdot p(\delta N'_V) \cdot \delta N'_V \cdot e^{\frac{|s|}{\tau}} \cdot d\delta N'_V = e^{\frac{|s|}{\tau}} \cdot \overline{\delta N'^2_V} \quad (5)$$

The variance of  $\delta N'_V$  is known since  $N'_V \cdot \Delta V \cdot \Delta E$  is a binomial distribution (there are  $N_t \cdot \Delta V \cdot \Delta E$  traps being occupied or empty):  $\overline{\delta N'^2_V} = \frac{N_t \cdot \Delta E \cdot f_t (1 - f_t)}{\Delta V}$ .

To find the PSD it is necessary to Fourier transform (5) (unilateral PSD):

$$S_{\delta N'_V}(\omega) = 2 \cdot \Re(\Re(s)) = \frac{1}{\Delta V} \cdot N_t \cdot f_t (1 - f_t) \cdot \Delta E \cdot \frac{4\tau}{1 + \omega^2 \tau^2} \quad (6)$$

$\omega = 2 \cdot \pi f$ . Integrating (6) in the  $z$  coordinate and in the energy:

$$S_{\Delta N'_V}(\omega) = \frac{1}{\Delta A} \int_{E_C}^{E_V} \int_0^{t_{ox}} N_t \cdot f_t (1 - f_t) \cdot \frac{4\tau}{1 + \omega^2 \tau^2} \cdot dz \cdot dE \quad (7)$$

Note that the integration boundaries in (7) are  $E_C, E_V$  (valence and conduction band energy) instead of  $\pm \infty$ . This classical approximation is supported by the fact that the product  $N_t f_t (1 - f_t)$  is usually sharply peaked. It is also supported from a physical perspective: consider an electron that gains extra energy interacting with a phonon and could tunnel to a trap with an energy  $E \geq E_C$ . This electron will encounter in the conduction band a sea of states with such energy and it is very unlikely that it would jump to the trap. Therefore, the probability of an electron tunneling to a trap is negligible outside the energy gap where it competes with a continuum of empty energy states at conduction ( $E \geq E_C$ ) or valence band ( $E \leq E_V$ ).

Classical approximations to solve (7) assume that  $\tau$  depends only on the distance  $z$ , and  $N_t, f_t$  on the energy. It is then possible to integrate (7) in the distance:

$$\int_0^{t_{ox}} \frac{\tau}{1+\omega^2\tau^2} dz = \frac{\lambda}{\omega} \left( \tan^{-1}(\omega\tau(t_{ox})) - \tan^{-1}(\omega\tau(0)) \right) \cong \frac{\lambda}{4f} \quad (8)$$

The last approximation is due to the high dispersion in values (7) and shows the classical  $1/f$  dependence of flicker noise. The integration in the energy of (7) can be carried out with a probability balance generalized to both electrons and holes. A detailed calculation is presented in [7] the result being:

$$\int_0^\infty N_t f_t (1-f_t) \cdot dE \approx N_t KT \quad (9)$$

The simplified result is that:

$$S_{N_t} = \frac{1}{\Delta A} N_t KT \lambda \cdot \frac{1}{f} = \frac{N_{ot}}{\Delta A} \cdot \frac{1}{f} \quad (10)$$

$N_{ot} = N_t KT \lambda$  in (10) is the equivalent density of oxide traps, a technology parameter to adjust.

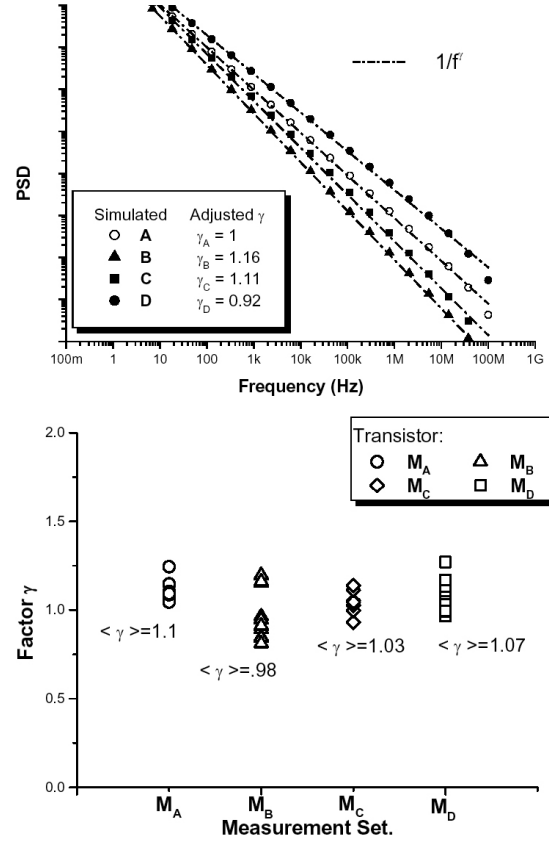
### A. The variations in $\gamma$ coefficient

It is a known fact that  $\gamma$  in (1) is not exactly 1. Variations in  $\gamma$  are attributed to a non-uniform distribution of traps inside the oxide [8]. At this stage  $\tau$  will still be considered as depending only on  $z$ . Re-writing (7):

$$S_{\Delta N_t}(\omega) = \frac{1}{\Delta A} \int_0^{t_{ox}} \eta(z) \cdot \frac{4\tau}{1+\omega^2\tau^2} \cdot dz \quad (11)$$

To evaluate the influence in  $\gamma$  of a non-uniform trap distribution along the oxide, some simulations of (11) were performed for the following cases: A)  $\eta(z)$  constant; B)  $\eta(z)$  positive exponential; C)  $\eta(z)$  linear; D)  $\eta(z)$  negative exponential. The result is shown at several frequencies in Figure 2.a.

The picture demonstrates that the model is still valid for A, B, C, D with different  $\gamma$  coefficients. In the plot of Figure 2.b, the adjusted values of  $\gamma$  for different measurements of flicker noise in MOS transistors are shown.



**Figure 2:** a) Simulated  $\gamma$  for different trap distributions. b) Adjusted  $\gamma$  for different measurements. M<sub>A</sub> is saturated NMOS, W/L=200/16; M<sub>B</sub> is saturated PMOS, W/L=200/16; M<sub>C</sub> is linear region NMOS, W/L=200/16; M<sub>D</sub> is saturated NMOS, W/L=40/12.

### 3. SIMULATION OF $1/f$ SWITCHED NOISE

A reduction in flicker noise PSD is expected in switched operation of the MOSFET [2,13]. Unfortunately, for the calculation of flicker noise in a switched MOS it was not possible to derive an analytical expression analogous to (5). Instead the autocorrelation was calculated using a transient simulation. In other words, for single or multiple traps, their state was simulated along time, using time steps  $dt$ , with selected statistical assumptions. In this section a general simulation framework for studying  $1/f$  switched noise is presented.

#### A. Simulation of Flicker noise in DC biased Transistors.

In deep sub-micron technologies it is possible to see the effect of a single trap usually referred as Random Telegraph Signal (RTS) (Figure 3.a). The deduction of the PSD of RTS can be calculated as in (6) but for a single trap. The result is a Lorentzian spectrum, flat for lower frequencies and decaying with 40 dB per decade starting at the frequency  $f_c$ :

$$S_{RTS}(f) = \frac{4C^2}{(\tau_c + \tau_e) \cdot [(2\pi f_c)^2 + (2\pi f)^2]}$$

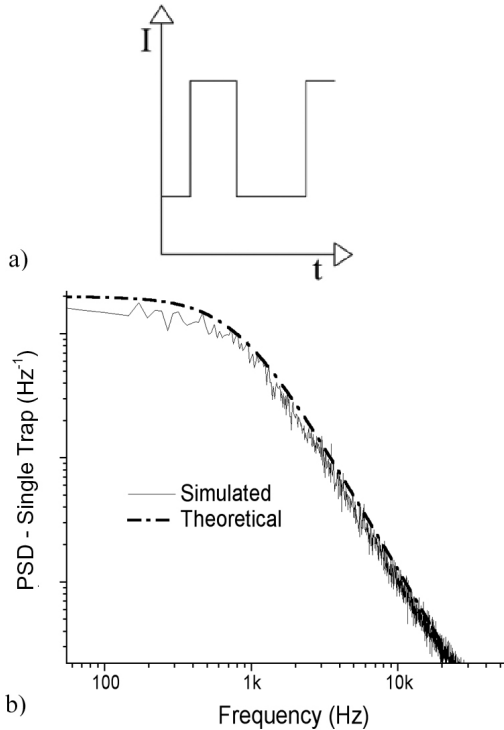
$$2\pi f_c = \frac{1}{\tau_c} + \frac{1}{\tau_e} \quad (12)$$

We shall denote  $\tau_c$  as the mean time before an electron is captured by the trap and  $\tau_e$  as the mean time before it is emitted. This time constants can be related to the probabilities seen in section II, with  $\tau_c = 1/c_0 n_S$  and  $\tau_e = 1/e_0$ . To simplify simulations it will be assumed  $\tau_c^{-1} = \tau_e^{-1} = \pi f_c$  as in [4]. A time-discrete model of a RTS was implemented in MATLAB. At every time step the probabilities of transition were calculated as follows:

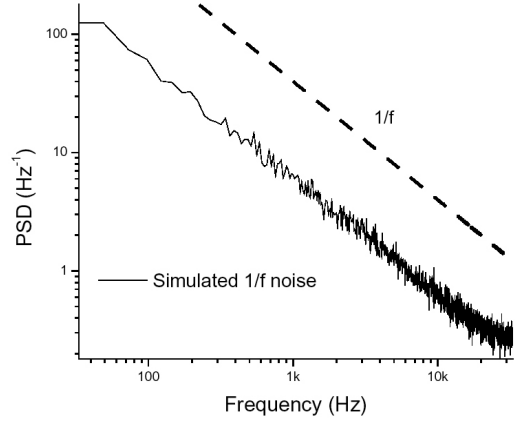
$$P_{capture} = \frac{T_S}{\tau_c}, \quad P_{emission} = \frac{T_S}{\tau_e} \quad (13)$$

Where  $T_S$  is the time step of our simulation. In Figure 3.b the simulated and theoretical PSD of a RTS with a corner frequency of 800 Hz are shown. This simulation was run 50 times and averaged to reduce error.

Analogous to (8), to simulate the effect of multiple traps with different time constants, all traps are assumed statistically independent of each other. The RTS generated by each trap can then be added to compute the total noise. According to [3] the distri-



**Figure 3:** a) Example of a RTS b) Simulated and Theoretical PSD of RTS with  $f_c = 800\text{Hz}$ .



**Figure 4:** Simulated  $1/f$  noise with 30 traps.

bution of  $f_c$ 's is log uniform, distributed between  $f_{cH}$  and  $f_{cL}$ , the highest and lowest  $f_c$  frequencies of the traps considered.

$$g(2\pi f_c) = \frac{4K_B T A t_{ox} N_t}{2\pi f_c \log\left(\frac{f_{cH}}{f_{cL}}\right)} \quad (14)$$

Where  $k_B$  is the Boltzmann constant,  $T$  is the absolute temperature and  $A$  the transistor area. This can be modeled by considering traps in logarithmic steps between  $f_{cH}$  and  $f_{cL}$  [9]. When multiple traps are considered, the  $1/f$  spectrum is obtained. Figure 4 shows a simulation of  $1/f$  noise. In this case 30 traps were simulated with  $f_{cH} = 23 \text{ KHz}$  and  $f_{cL} = 1 \text{ Hz}$ , and  $T_S = 0.01 \text{ ms}$ .

If only few traps and with just two different time constants are simulated, the results are similar to the ones presented in [11].

## B. Model for switching $1/f$ noise

When dealing with switched transistors, the periodically varying effect of turning 'on' and 'off' the transistor must be considered. When the  $V_{GS}$  voltage is reduced, the carrier density in the channel is reduced as well, and this changes the probabilities of capturing and emitting electrons by the traps.

When  $V_{GS} = 0$ , only a few conducting electrons are present in the channel and the probability of one of them being captured by a trap is negligible. In our simulation, we will consider that no electron will be captured when the transistor is in the 'off' state. On the other hand, the probability of emission of an electron from a trap will increase. We will take into consideration this increase with a factor,  $m$ , as follows [4]:

$$P_{OFFcapture} = 0$$

$$P_{OFFemission} = m^* P_{OFFemission} \quad (15)$$

Equation (15) is similar to the method of van der Wel et al [4] but simpler because no electron is captured during the ‘off’ state.

The work by Tian and El Gamal [3] uses the same procedure but with  $m = \infty$ . This model predicts that flicker noise PSD will remain constant at frequencies lower than the switching frequency. But reported measurements [2,13] show that although noise is reduced its PSD still resembles  $1/f$  at lowest frequencies.

In Figure 5, different simulations with different values of  $m$  are presented. The reduction of the  $1/f$  noise can thus be explained, and the  $m$  parameter can be fitted with experimental data. The simulations were conducted with the same 30 traps of Figure 4, and with a switching frequency of 10 KHz, 50% duty cycle.

### C. Resurgence of $1/f$ noise for lower frequencies

In the last subsection the reduction of flicker noise was explained but the resurgence of  $1/f$  spectrum for even lower frequencies reported in [2,13] was not. The assumption in the previous section was that the emission probabilities of all the traps are affected by variations of  $V_{GS}$  in the same way. That is, the  $m$  factor is the same for all traps. There is no reason for this to be so, and a reasonable hypothesis is to assume that the traps which are farther from the channel (and so with a lower  $f_c$ ) will be less affected by changes in its voltage. In the simulation showed in Figure 6, a simple, but different distribution of values of  $m$  was selected:

$$m(f_c) = \begin{cases} 100 & f_c > K \\ 10 & f_c < K \end{cases} \quad (16)$$

With  $K$  selected in the simulation of Figure 6, so that only 5 traps will be the less affected. The result of this new simulation shows the behavior of  $1/f$  noise

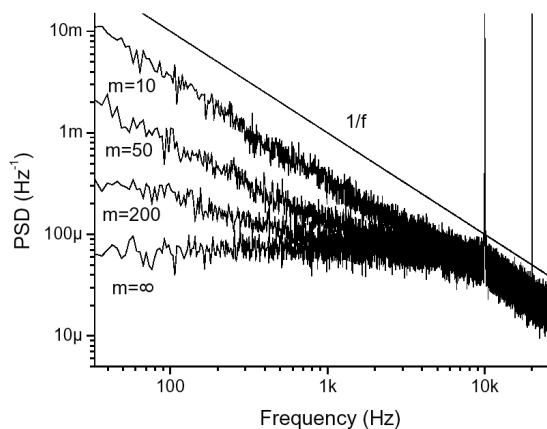


Figure 5: Different simulation of  $1/f$  switched noise showing the effect of varying  $m$ .

at lower frequencies. Although the selected distribution is quite arbitrary, it demonstrates that a physical model taking into account different  $m$  factors leads to results that can explain observed measurements.

### D. Duty Cycle dependence

Another interesting effect to investigate by means of simulations, is the reduction of flicker noise while varying the duty cycle. It is known that this reduction of  $1/f$  noise is more than one half if the switching is done with a duty cycle of 50%. In figure 7, several simulations for a single trap with  $f_c = 48$  Hz and different values of duty cycle, are shown.

The plot shows the normalized PSD of each trap (normalization means that each simulated PSD was multiplied by the inverse of the duty cycle) allowing the difference in shape of the plots to be observed. The reduction is greater as the time the transistor is in the ‘on’ state is reduced. This simulation was conducted on the conditions of figure 4 with  $m = 200$ , large enough to make this effect clear.

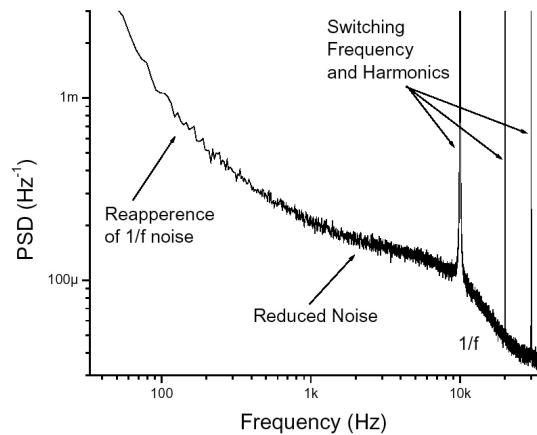


Figure 6: Simulation using different  $m$  factors for different traps. The resurgence of  $1/f$  noise at lower frequencies can be seen.

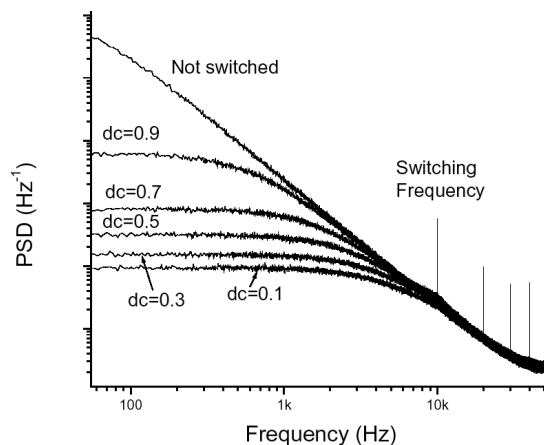


Figure 7: Reduction of  $1/f$  noise for different duty cycles (dc).

#### 4. CONCLUSIONS

An explicit deduction for flicker noise PSD was presented using SRH statistics and the autocorrelation formalism in the case of a DC biased transistor. The fluctuation of the  $\gamma$  coefficient originated by non-uniform trap spatial distribution was investigated.

A general simulation framework for studying flicker noise under switched bias conditions was presented. The case of a single trap (RTS) was shown, investigating also the effect of varying the duty cycle of switching. The simulation of several simultaneous traps led to the usual  $1/f$  spectrum.

Using the same simulation tools, the impact of considering different behavior for emission probabilities of the traps along the oxide while switching, was studied. To model the effect, a space-dependant  $m$  factor relating emission probabilities during 'on' and 'off' state was assumed. The result of simulations allowed the observation, at the lowest frequencies, of an increasing PSD resembling the original  $1/f$  spectrum. This behavior has been observed in previously reported measurements however it is addressed by few existing switched MOSFET flicker noise models, and its explanation not widely accepted.

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